Non-unitary HD gravity classically equivalent to Einstein gravity

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Runaway solutions can be avoided in fourth order gravity by a doubling of the matter operator algebra with a symmetry constraint with respect to the exchange of observable and hidden degrees of freedom together with the change in sign of the ghost and the dilaton fields. The theory is classically equivalent to Einstein gravity, while its non-unitary Newtonian limit is compatible with the wavelike properties of microscopic particles and the classical behavior of macroscopic bodies, as well as with a trans-Planckian regularization of collapse singularities. A unified reading of ordinary and black hole entropy emerges as entanglement entropy with hidden degrees of freedom.

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Although higher derivative (HD) gravity has long been popular as a natural generalization of Einstein gravity [1–3], since "perturbation theory for gravity ... requires higher derivatives in the free action" [4], already on the classical level it is unstable due to negative energy fields giving rise to runaway solutions [4]. On the quantum level an optimistic conclusion as to unitarity is that "the S-matrix will be nearly unitary [1]" [4].

A way out of the so called information loss paradox [5,6] of black hole physics [7] may be precisely a fundamental non-unitarity [8–12]: "For almost any initial quantum state, one would expect ... a nonvanishing probability for evolution from pure states to mixed states" [10]. Though such an evolution is incompatible with a cherished principle of quantum theory,

the crucial issue is to see if it necessarily gives rise to a loss of coherence or to violations of energy-momentum conservation so large as to be incompatible with ordinary laboratory physics [8–10,12], as guessed for Markovian effective evolution laws [8,9]. However one expects that a law modeling black hole formation and evaporation, far from being local in time, should retain a long term "memory" [10,12].

Here a specific non-unitary realization of HD gravity is shown to be classically stable, as well as compatible with the wavelike properties of microscopic particles and with the assumption of a gravity-induced emergence of classicality [13–20]. Moreover it leads to the reading of the thermodynamical entropy of a closed system as von Neumann entropy, or equivalently as entanglement entropy with hidden degrees of freedom [10,12], which allows, in principle, to overcome the dualistic nature of the notions of ordinary and Bekenstein-Hawking (B-H) entropy [21]. To be specific, the B-H entropy [21] may be identified with the von Neumann entropy of the collapsed matter, or equivalently with the entanglement entropy between matter and hidden degrees of freedom, both close to the smoothed singularity. In fact the model seems to give clues for the elimination of singularities on a trans-Planckian scale. Parenthetically we are encouraged in our extrapolations by the success of inflationary models, implicitly referring to these scales [22]. This reading of B-H entropy may appear rather natural, as the high curvature region is where new physics is likely to emerge. However, in passing from the horizon [12], where quantum field theory in curved space-times is expected to work, to the region close to the classical singularity, in the absence of a full theory of quantum gravity, we have to rely on heuristic arguments and some guessing work, which we intend to show can be carried out by rather natural assumptions. This reading, however, is corroborated by the attractive features of the Newtonian limit of the model.

Similarly to Ref. [4], we consider first a simpler fourth order theory for a scalar field ϕ, which has the same ghostly behavior as HD gravity. Its action

$$S = \int d^4x \left[-\phi \Box \left(\Box - \mu^2 \right) \phi / 2 - \lambda \phi^4 + \alpha \psi^\dagger \psi \phi \right] + S_{mat} \left[\psi^\dagger, \psi \right] \tag{1}$$

includes a matter action S_{mat} and an interaction with matter, where $\psi^\dagger \psi$ is a shorthand

notation for a quadratic scalar expression in matter fields. Defining

$$\phi_1 = \left(\Box - \mu^2\right)\phi/\mu,\ \phi_2 = \Box\phi/\mu, \tag{2}$$

the action can be rewritten as

$$\begin{aligned} &S\left[\phi_1, \phi_2, \psi^\dagger, \psi\right] \\ &= \int d^4x \left[\frac{1}{2}\phi_1\Box\phi_1 - \frac{1}{2}\phi_2\left(\Box - \mu^2\right)\phi_2 - \lambda\left(\frac{\phi_2 - \phi_1}{\mu}\right)^4 + \frac{\alpha}{\mu}\psi^\dagger\psi\left(\phi_2 - \phi_1\right)\right] + S_{mat}\left[\psi^\dagger, \psi\right]. \end{aligned} \tag{3}$$

The quadratic term in ϕ_2 has the wrong sign, which classically means that the energy of this field is negative. Due to the presence of interactions, energy can flow from negative to positive energy degrees of freedom, and one can have runaway solutions [4].

In this model there is a cancellation of all self-energy and vertex infinities coming from the $\psi^\dagger\psi\phi$ interaction, owing to the difference in sign between ϕ_1 and ϕ_2 propagators. This feature, "analogous to the Pauli-Villars regularization of other field theories" [2], is responsible for the improved ultraviolet behavior in HD gravity [2]. A key feature of the non-interacting theory ($\lambda = \alpha = 0$), making it classically viable, can be considered to be its symmetry under the transformation $\phi_2 \longrightarrow -\phi_2$, by which symmetrical initial conditions with $\phi_2 = 0$ produce symmetrical solutions. If one symmetrizes the Lagrangian (3) as it is, in order to extend this symmetry to the interacting theory, this eliminates the interaction between the ghost field and the matter altogether and, with it, the mentioned cancellations. A possible procedure to get a symmetric action while keeping cancellations is suggested by previous attempts [4] and by the information loss paradox [10], both pointing to a non-unitary theory with hidden degrees of freedom. In particular the most natural way to make the hidden degrees of freedom "not ... available as either a net source or a sink of energy" [10] is to constraint them to be a copy of observable ones. Accordingly we introduce a (meta-)matter algebra that is the product of two copies of the observable matter algebra, respectively generated by the $\psi^\dagger, \psi$ and $\tilde{\psi}^\dagger, \tilde{\psi}$ operators, and a symmetrized action

$$S_{Sym} = \left\{S\left[\phi_1, \phi_2, \psi^\dagger, \psi\right] + S\left[\phi_1, -\phi_2, \tilde{\psi}^\dagger, \tilde{\psi}\right]\right\}/2, \tag{4}$$

which is invariant under the symmetry transformation

$$\phi_1 \longrightarrow \phi_1, \quad \phi_2 \longrightarrow -\phi_2, \quad \psi \longrightarrow \tilde{\psi}, \quad \tilde{\psi} \longrightarrow \psi. \tag{5}$$

If the symmetry constraint is imposed on states $|\Psi\rangle$, i.e. the state space is restricted to those states that are generated from the vacuum by symmetrical operators, then

$$\langle \Psi | F\left[\phi_2, \psi^\dagger, \psi\right] |\Psi\rangle = \langle \Psi | F\left[-\phi_2, \tilde{\psi}^\dagger, \tilde{\psi}\right] |\Psi\rangle \quad \forall F. \tag{6}$$

The allowed states do not give a faithful representation of the original algebra, which is then larger than the observable algebra. In particular they cannot distinguish between $F\left[\psi^\dagger, \psi\right]$ and $F\left[\tilde{\psi}^\dagger, \tilde{\psi}\right]$, by which the $\tilde{\psi}$ operators are referred to hidden degrees of freedom [10]. On a classical level ψ and $\tilde{\psi}$ are identified, the ϕ_2 field vanishes and the classical constrained action is that of an ordinary second order scalar theory interacting with matter:

$$S_{Cl} = \int d^4x \left[\phi_1 \Box \phi_1/2 - \lambda \left(\phi_1/\mu\right)^4 - \alpha \phi_1 \psi^\dagger \psi/\mu\right] + S_{mat}\left[\psi^\dagger, \psi\right]. \tag{7}$$

Consider now the action of a fourth order theory of gravity including matter [2]

$$\begin{aligned} S &= S_G\left[g_{\mu\nu}\right] + S_{mat}\left[g_{\mu\nu}, \psi^\dagger, \psi\right] \\ &= -\int d^4x \sqrt{-g}\left[\alpha R_{\mu\nu} R^{\mu\nu} - \beta R^2 + R/\left(16\pi G\right)\right] + \int d^4x \sqrt{-g} L_{mat}, \end{aligned} \tag{8}$$

where L_{mat} denotes the matter Lagrangian density in a generally covariant form. In terms of the contravariant metric density

$$\sqrt{32\pi G} h^{\mu\nu} = \sqrt{-g} g^{\mu\nu} - \eta^{\mu\nu}, \tag{9}$$

the Newtonian limit of the static field is

$$h^{00} \sim 1/r + e^{-\mu_0 r}/(3r) - 4e^{-\mu_2 r}/(3r), \tag{10}$$

where $\mu_0 = [32\pi G(3\beta - \alpha)]^{-1/2}$, $\mu_2 = [16\pi G\alpha]^{-1/2}$ [2]. From Stelle's linearized analysis [2], the first term in Eq. (10) corresponds to the graviton, the second one to a massive scalar and

the third one to a negative energy spin-two field. In fact, in analogy with Eq. (2), one can introduce a transformation from $g_{\mu\nu}$ to a new metric tensor $\bar{g}_{\mu\nu}$, a scalar field χ dilatonically coupled to $\bar{g}_{\mu\nu}$ and a spin-two field $\phi_{\mu\nu}$, this transformation leading to the second order form of the action [23]. To be specific, referring to Ref. [23] (see Eq. (6.9) apart from the matter term), the action (8) becomes the sum of the Einstein-Hilbert action S_{EH} for $\bar{g}_{\mu\nu}$, an action S_{gh} for $\phi_{\mu\nu}$ and χ coupled to $\bar{g}_{\mu\nu}$, and a matter action S_{mat}, with $g_{\mu\nu}$ expressed in terms of $\bar{g}_{\mu\nu}$, $\phi_{\mu\nu}$ and χ (replacing $g_{\mu\nu}$ by $e^{\chi}g_{\mu\nu}$ in Eq. (4.12) in Ref. [23]):

$$\begin{aligned} &S\left[\bar{g}_{\mu\nu}, \phi_{\mu\nu}, \chi, \psi^{\dagger}, \psi\right] \\ &= S_{EH}\left[\bar{g}_{\mu\nu}\right] + S_{gh}\left[\bar{g}_{\mu\nu}, \phi_{\mu\nu}, \chi\right] + S_{mat}\left[g_{\mu\nu}(\bar{g}_{\sigma\tau}, \phi_{\sigma\tau}, \chi), \psi^{\dagger}, \psi\right]. \end{aligned} \tag{11}$$

In S_{gh} the quadratic part in $\phi_{\mu\nu}$ has the wrong sign [23]. One could symmetrize the action with respect to the transformation $\phi_{\mu\nu} \to -\phi_{\mu\nu}$, but this would eliminate the repulsive term in Eq. (10), which below plays a role in avoiding the singularity in gravitational collapse. Like for the toy model, we double the matter algebra and define the symmetrized action

$$S_{Sym} = \left\{ S\left[\bar{g}_{\mu\nu}, \phi_{\mu\nu}, \chi, \psi^{\dagger}, \psi\right] + S\left[\bar{g}_{\mu\nu}, -\phi_{\mu\nu}, -\chi, \tilde{\psi}^{\dagger}, \tilde{\psi}\right] \right\}/2, \tag{12}$$

which is symmetric under the transformation

$$\bar{g}_{\mu\nu} \longrightarrow \bar{g}_{\mu\nu}, \;\; \phi_{\sigma\tau} \to -\phi_{\sigma\tau}, \; \chi \longrightarrow -\chi, \; \psi \longrightarrow \tilde{\psi}, \; \tilde{\psi} \longrightarrow \psi. \tag{13}$$

If only symmetric states are allowed, the $\tilde{\psi}$ operators denote hidden degrees of freedom, as

$$\langle\Psi| F\left[\bar{g}_{\mu\nu}, \phi_{\mu\nu}, \chi, \psi^{\dagger}, \psi\right] |\Psi\rangle = \langle\Psi| F\left[\bar{g}_{\mu\nu}, -\phi_{\mu\nu}, -\chi, \tilde{\psi}^{\dagger}, \tilde{\psi}\right] |\Psi\rangle \quad \forall F. \tag{14}$$

On a classical level ψ and $\tilde{\psi}$ are identified, while the $\phi_{\mu\nu}$ and χ fields vanish and the classical constrained action is that of ordinary matter coupled to ordinary gravity:

$$S_{Cl}\left[\bar{g}_{\mu\nu}, \psi^{\dagger}, \psi\right] = S_{EH}\left[\bar{g}_{\mu\nu}\right] + S_{mat}\left[\bar{g}_{\mu\nu}, \psi^{\dagger}, \psi\right], \tag{15}$$

as $S_{gh}\left[\bar{g}_{\mu\nu}, 0, 0\right] = 0$ (Eq. (6.9) in Ref. [23]) and $g_{\mu\nu}(\bar{g}_{\sigma\tau}, 0, 0) = \bar{g}_{\sigma\tau}$ (Eq. (4.12) in Ref. [23] with $e^{\chi}g_{\mu\nu}$ replacing $g_{\mu\nu}$).

After the elimination of classical runaway solutions, a further natural step in assessing the consistency of the theory is the study of its implications for ordinary laboratory physics. Consider the Newtonian limit with non-relativistic meta-matter and instantaneous action at a distance. By Eq. (12), we see that the interactions due to $\bar{g}_{\mu\nu}$ are always attractive, whereas those due to $\phi_{\mu\nu}$ are repulsive within observable and within hidden meta-matter, as shown by the minus sign in Eq. (10), and are otherwise attractive, as the ghostly character is offset by the difference in sign in its coupling with observable and hidden meta-matter; the reverse is true for the scalar field χ. The corresponding (meta-)Hamiltonian is

$$\begin{aligned} H_G = H_0[\psi^\dagger,\psi] - \frac{G}{4}\sum_{j,k} m_j m_k \int dxdy \frac{:\psi_j^\dagger(x)\psi_j(x)\psi_k^\dagger(y)\psi_k(y):}{|x-y|}\left(1+\frac{e^{-\mu_0|x-y|}}{3}-\frac{4e^{-\mu_2|x-y|}}{3}\right) \\ +H_0[\tilde{\psi}^\dagger,\tilde{\psi}] - \frac{G}{4}\sum_{j,k} m_j m_k \int dxdy \frac{:\tilde{\psi}_j^\dagger(x)\tilde{\psi}_j(x)\tilde{\psi}_k^\dagger(y)\tilde{\psi}_k(y):}{|x-y|}\left(1+\frac{e^{-\mu_0|x-y|}}{3}-\frac{4e^{-\mu_2|x-y|}}{3}\right) \\ -\frac{G}{2}\sum_{j,k} m_j m_k \int dxdy \frac{\psi_j^\dagger(x)\psi_j(x)\tilde{\psi}_k^\dagger(y)\tilde{\psi}_k(y)}{|x-y|}\left(1-\frac{e^{-\mu_0|x-y|}}{3}+\frac{4e^{-\mu_2|x-y|}}{3}\right) \end{aligned} \tag{16}$$

acting on the product $F_\psi \otimes F_{\tilde{\psi}}$ of the Fock spaces of (the non-relativistic counterparts of) ψ and $\tilde{\psi}$. Two couples of meta-matter operators $\psi_j^\dagger, \psi_j$ and $\tilde{\psi}_j^\dagger, \tilde{\psi}_j$ appear for every particle species and spin component, while m_j is the mass of the j-th species and H_0 is the gravitationless matter Hamiltonian. The $\tilde{\psi}$ operators obey the same statistics as the corresponding operators ψ, while $[\psi,\tilde{\psi}]_- = [\psi,\tilde{\psi}^\dagger]_- = 0$. Tracing out $\tilde{\psi}$ from a symmetrical meta-state evolving according to the unitary meta-dynamics generated by H_G results in a non-Markov non-unitary physical dynamics for the ordinary matter algebra [24].

Considering, for simplicity, particles of one and the same species, the time derivative of the matter canonical momentum in a space region Ω in the Heisenberg picture reads

$$\begin{aligned} \frac{d\overrightarrow{p}_\Omega}{dt} = -i\hbar\frac{d}{dt}\int_\Omega dx\psi^\dagger(x)\nabla\psi(x) \equiv \left.\frac{d\overrightarrow{p}_\Omega}{dt}\right|_{G=0} + \vec{F}_G = -\frac{i}{\hbar}\left[\overrightarrow{p}_\Omega, H_0[\psi^\dagger,\psi]\right] \\ +\frac{G}{2}m^2\int_\Omega dx\psi^\dagger(x)\psi(x)\nabla_x\int_{R^3} dy\frac{\tilde{\psi}^\dagger(y)\tilde{\psi}(y)}{|x-y|}\left(1-\frac{e^{-\mu_0|x-y|}}{3}+\frac{4e^{-\mu_2|x-y|}}{3}\right) \\ +\frac{G}{2}m^2 :\int_\Omega dx\psi^\dagger(x)\psi(x)\nabla_x\int_{R^3} dy\frac{\psi^\dagger(y)\psi(y)}{|x-y|}\left(1+\frac{e^{-\mu_0|x-y|}}{3}-\frac{4e^{-\mu_2|x-y|}}{3}\right): . \end{aligned} \tag{17}$$

The expectation of the gravitational force can be written as

$$
\begin{aligned}
\left\langle \vec{F}_G \right\rangle =& \frac{G}{2} m^2 \left\langle \int_\Omega dx \psi^\dagger(x)\psi(x) \nabla_x \int_\Omega dy \frac{\tilde{\psi}^\dagger(y)\tilde{\psi}(y)}{|x-y|} \left(1 - \frac{e^{-\mu_0|x-y|}}{3} + \frac{4e^{-\mu_2|x-y|}}{3}\right)\right\rangle \\
&+ \frac{G}{2} m^2 \left\langle \int_\Omega dx \psi^\dagger(x)\psi(x) \nabla_x \int_{R^3\setminus\Omega} dy \frac{\tilde{\psi}^\dagger(y)\tilde{\psi}(y)}{|x-y|} \left(1 - \frac{e^{-\mu_0|x-y|}}{3} + \frac{4e^{-\mu_2|x-y|}}{3}\right)\right\rangle \\
&+ \frac{G}{2} m^2 \left\langle : \int_\Omega dx \psi^\dagger(x)\psi(x) \nabla_x \int_\Omega dy \frac{\psi^\dagger(y)\psi(y)}{|x-y|} \left(1 + \frac{e^{-\mu_0|x-y|}}{3} - \frac{4e^{-\mu_2|x-y|}}{3}\right) : \right\rangle \\
&+ \frac{G}{2} m^2 \left\langle \int_\Omega dx \psi^\dagger(x)\psi(x) \nabla_x \int_{R^3\setminus\Omega} dy \frac{\psi^\dagger(y)\psi(y)}{|x-y|} \left(1 + \frac{e^{-\mu_0|x-y|}}{3} - \frac{4e^{-\mu_2|x-y|}}{3}\right)\right\rangle , \quad (18)
\end{aligned}
$$

where, on allowed states, the first term vanishes for the antisymmetry of the kernel $\nabla_x \left[\left(1 - e^{-\mu_0|x-y|}/3 + 4e^{-\mu_2|x-y|}/3\right) / |x-y| \right]$ and the symmetry of the state, while the third one vanishes just as a consequence of the antisymmetry of the corresponding kernel. We can approximate $\left\langle \psi^\dagger(x)\psi(x)\tilde{\psi}^\dagger(y)\tilde{\psi}(y) \right\rangle$ and $\left\langle \psi^\dagger(x)\psi(x)\psi^\dagger(y)\psi(y) \right\rangle$ respectively with $\left\langle \psi^\dagger(x)\psi(x) \right\rangle \left\langle \tilde{\psi}^\dagger(y)\tilde{\psi}(y) \right\rangle$ and $\left\langle \psi^\dagger(x)\psi(x) \right\rangle \left\langle \psi^\dagger(y)\psi(y) \right\rangle$, as $x \in \Omega$ and $y \in R^3\setminus\Omega$. Finally, as $\left\langle \tilde{\psi}^\dagger(y)\tilde{\psi}(y) \right\rangle = \left\langle \psi^\dagger(y)\psi(y) \right\rangle$, we get

$$
\left\langle \vec{F}_G \right\rangle \simeq G m^2 \int_\Omega dx \left\langle \psi^\dagger(x)\psi(x) \right\rangle \nabla_x \int_{R^3\setminus\Omega} dy \left\langle \psi^\dagger(y)\psi(y) \right\rangle / |x-y| , \quad (19)
$$

as for the traditional Newton interaction between observable degrees of freedom only, consistently with the classical equivalence of the original theory to Einstein gravity.

As to the quantum aspects of the present Newtonian model, a closely related model was analyzed in Ref.s [24]. Actually, if in Ref. [24] $H[\psi^\dagger, \psi]$ is meant to be the sum of $H_0[\psi^\dagger, \psi]$ and the normal ordered interaction within observable matter in Eq. (16) above, and analogously for the hidden meta-matter, the two models differ only for the kernel in the interaction between observable and hidden meta-matter. The main results, which stay qualitatively unchanged, are the following. For the center of mass wave function of a homogeneous body of mass M and linear dimensions R, effective gravitational self-interactions lead to a localization length $\Lambda \sim (\hbar^2 R^3/GM^3)^{1/4}$, as soon as it is small with respect to R. This produces a rather sharp threshold that, for ordinary densities $\sim 10^{24} m_p/cm^3$, where m_p denotes the proton mass, is around $10^{11} m_p$, below which the effects of the effective gravitational self-interactions are irrelevant [24]. A localized state slowly evolves, with times

$\sim 10^3\,\mathrm{sec}$, for ordinary densities, into a delocalized ensemble of localized states [24], this entropic spreading replacing the wave function spreading of ordinary QM and the unphysical stationary localized states of the nonlinear unitary Schroedinger-Newton (S-N) model [25–28]. As to an unlocalized pure state of a body above threshold, it rapidly gets localized, within times $\sim 10^{20}(M/m_p)^{-5/3}\,\mathrm{sec}$, under reasonable assumptions on the initial state, into such an ensemble [24]. This gives a well-defined dynamical model for gravity-induced decoherence, to be compared with purely numerological estimates [13–20] and which allows us to address physically relevant problems, like the characterization of gravitationally decoherence free states of the physical operator algebra [29]. It is worthwhile to remark that, in spite of the presence of the masses $\mu_i, i = 0, 2$ (actually $\hbar c\mu_i$), the Newtonian limit has, for all practical purposes, no free parameter, as to ordinary laboratory physics, if as usual they are assumed to be of the order of the Planck mass, which is equivalent to take the limit $\mu_i \to \infty$.

If the traditional Hamiltonian includes the Newton interaction, there are extremely small violations of energy conservation, as only the meta-Hamiltonian H_G is strictly conserved. These fluctuations are consistent with the assumption that an eigenstate of the traditional Hamiltonian may evolve towards a microcanonical mixed state with an energy dispersion around the original energy, which, though irrelevant on a macroscopic scale, paves the way for the possibility that the thermodynamic entropy of a closed system may be identified with its von Neumann entropy [11]. This is not irrelevant, if "...in order to gain a better understanding of the degrees of freedom responsible for black hole entropy, it will be necessary to achieve a deeper understanding of the notion of entropy itself. Even in flat space-time, there is far from universal agreement as to the meaning of entropy – particularly in quantum theory – and as to the nature of the second law of thermodynamics" [12]. Of course the reversibility of the unitary meta-dynamics makes entropy decrease conceivable too [30], so that a derivation of the entropy-growth for a closed system in the present context must have recourse to the choice of suitable initial conditions, like unentanglement between the observable and the hidden algebras [11]. While the assumption of special initial conditions dates back to Boltzmann, only a non-unitary dynamics makes it a viable starting point, within a

quantum context, for the microscopic derivation of the second law of thermodynamics, in terms of von Neumann entropy, without renouncing strict isolation [31].

Emboldened by the mentioned bonuses coming from the Newtonian limit of our model, we now try to apply it to gravitational collapse. First evaluate within our model the linear dimension of a collapsed matter lump, replacing the classical singularity. In order to do that we boldly use Eq. (16) for lengths smaller than μ_0^{-1} and μ_2^{-2}, namely in the limit $\mu_0, \mu_2 \to 0$. This corresponds to the replacement of our meta-Hamiltonian with the meta-Hamiltonian in Ref. [24], where there is no gravitational interaction within observable and within hidden matter, while there is a Newton interaction between observable and hidden matter. This interaction is effective in lowering the gravitational energy of a matter lump as far as the localization length $\Lambda = (\hbar^2 R^3/GM^3)^{1/4}$ is fairly smaller than the lump radius R [24]. The highest possible density then corresponds roughly to $\Lambda = R$, namely to

$$R = \hbar^2 / \left(GM^3\right). \tag{20}$$

In fact, below the localization threshold, only the interactions within observable (and hidden meta-) matter are effective in collapsing matter, but, in the considered limit $\mu_0, \mu_2 \to 0$, they vanish. This parenthetically shows that the following discussion depends crucially, not only on the doubling of the matter degrees of freedom, but also on the inclusion of the repulsive interactions of HD gravity.

As to the space-time geometry, the Schwarzschild metric in ingoing Eddington-Finkelstein coordinates (v, r, θ, ϕ) covers the two regions of the Kruskal maximal extension that are relevant to gravitational collapses [32]:

$$ds^2 = -\left[1 - 2MG/\left(rc^2\right)\right] dv^2 + 2dr dv + r^2 \left[d\theta^2 + \sin^2\theta d\phi^2\right]. \tag{21}$$

If, in the region beyond the horizon we put $x = v - \int dr \left[1 - 2MG/\left(rc^2\right)\right]^{-1}$, then

$$ds^2 = \left[1 - 2MG/\left(rc^2\right)\right]^{-1} dr^2 - \left[1 - 2MG/\left(rc^2\right)\right] dx^2 + r^2 \left[d\theta^2 + \sin^2\theta d\phi^2\right], \tag{22}$$

where we see that beyond the horizon r may be regarded as a time variable [32].

If we trust (20) as the minimal length involved in the collapse, we are led to assume that a full theory of quantum gravity should include a mechanism regularizing the singularity at $r = 0$ by means of that minimal length. In particular, to characterize the region occupied by the collapsed lump, consider that for time-like geodesics at constant θ and ϕ one can show that $|dx/dr| \sim r^{3/2}$ as $r \to 0$. This implies that, the x coordinate difference Δx of two material points has a well defined limit as $r \to 0$, by which it is natural to assume that the x width of the collapsed matter lump is $\Delta x \sim R$. As to the apparent inconsistency of matter occupying just a finite Δx interval with $\partial/\partial x$ being a Killing vector, one should expect on sub-Planckian scales substantial quantum corrections to the Einstein equations that the model gives on a classical level (15), with the dilaton and the ghost fields, though vanishing in the average, playing a crucial role. On the other hand we are proceeding according to the usual assumption, or fiction, of QM on the existence of a global time variable, at least in the region swept by the lump. In fact the most natural way to regularize (22) is to consider it as an approximation for $r > R$ of a regular metric, whose coefficients for $r \to 0$ correspond to the ones in (22) with $r = R$, in which case there is no obstruction in extending the metric to $r < 0$, where taking constant coefficients makes $\partial/\partial r$ a time-like Killing vector. As a consequence, the relevant space metric in the region swept by the collapsed lump is

$$ds^2_{SPACE} \sim 2MG/\left(Rc^2\right) dx^2 + R^2 \left[d\theta^2 + \sin^2\theta d\phi^2\right]. \tag{23}$$

The volume of the collapsed matter lump is then:

$$V \sim R^2 \Delta x \sqrt{MG/\left(Rc^2\right)} = \left[\hbar^2/\left(GM^3\right)\right]^{5/2} \sqrt{MG/c^2} = \hbar^5 M^{-7}/\left(G^2 c\right). \tag{24}$$

According to the above view, thermodynamical equilibrium is reached, due to the gravitational interaction generating entanglement between the observable and hidden meta-matter, by which the matter state is a microcanonical ensemble corresponding to the energy

$$E = Mc^2 + GM^2/R = Mc^2 + GM^2\left[GM^3/\hbar^2\right] \sim G^2 M^5/\hbar^2, \quad if \quad M \gg M_P, \tag{25}$$

where $M_P = \sqrt{\hbar c/G}$ is the Planck mass, and to the energy density

$$\varepsilon = E/V \sim G^4 c M^{12}/\hbar^7. \tag{26}$$

We first treat the collapsed lump as a three-dimensional bulk in spite of the huge dilation factor in the x direction. Since this energy density corresponds to a very high temperature, not to be mistaken for the Hawking temperature, the matter can be represented by massless fields, whose equilibrium entropy is given by

$$S \sim \left(K_B/\ \hbar^{3/4} c^{3/4}\right) \varepsilon^{3/4} V = GM^2 K_B/\left(\hbar c\right). \tag{27}$$

Of course this result can be trusted at most for its order of magnitude, the uncertainty in the number of species being just one part of an unknown numerical factor. With this proviso, common to other approaches [12], Eq. (27) agrees with B-H entropy. As this result contains four dimensional constants, it is more than pure numerology: from a dimensional viewpoint one could replace the dimensionless quantity $GM^2/\hbar c$ in Eq. (27) by any arbitrary function of $GM^2/\hbar c$. For instance, if we ignored that the space geometry near the smoothed singularity is not euclidean and then we assumed that $V \sim R^3$, we would get the entropy to be independent from the black hole mass, since the increase of the entropy density with the mass would be offset by the shrinking of the volume.

One could object, against the above derivation, that the collapsed matter lump, for the presence of the dilation factor along the x direction, is more like a one-dimensional string-like structure of transverse dimensions $\sim R$ along θ and ϕ and length $L = (MG/Rc^2)^{1/2} R \gg R$. If we treat it like this, for the linear density of entropy we have $s \propto \varepsilon^{1/2}$ and for the length $L \propto M^{-1}$, by which $S = Ls \propto L(E/L)^{1/2} \propto M^2$, which agrees with the previous result and with B-H entropy.

Finally, if we give for granted that a future theory of quantum gravity will account for black hole evaporation, we can connect the temperature

$$T \sim \sqrt[4]{\varepsilon h^3 c^3}/K_B \sim cGM^3/K_B \hbar \tag{28}$$

of our collapsed matter lump with the temperature of the radiation at infinity. If we model radiation by massless fields, emitted for simplicity at a constant temperature as we are in-

terested just in orders of magnitude, this temperature is defined in terms of he ratio E_∞/S_∞ of its energy E_∞ and its entropy S_∞. It is natural to assume that, "once" thermodynamical equilibrium is reached due to the highly non-unitary dynamics close to the classical singularity, no entropy production occurs during evaporation, by which $S_\infty = S$. Then, if $E_\infty = Mc^2$ is the energy of the total Hawking radiation spread over a very large space volume, its temperature agrees with Hawking temperature, i.e.

$$T_\infty = (E_\infty/E)\, T \sim \left(c^3\hbar/MGK_B\right). \tag{29}$$

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